

Math 55 Quiz 7 DIS 105

Name: _____

11 Apr 2022

1. Use Chebyshev's inequality to find an upper bound on the probability that the number of tails that come up when a fair coin is tossed n times deviates from the mean by more than $5\sqrt{n}$. [4 points]

Let X be the number of tails that come up when a fair coin is tossed n times. Using the formula for Bernoulli trials, $V(X) = n \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{n}{4}$, so using Chebyshev's inequality, $P(|X - E(X)| > 5\sqrt{n}) < \frac{V(X)}{(5\sqrt{n})^2} = \frac{1}{100}$.

2. (a) Find a recurrence relation for the number of ways to climb n stairs if the person climbing the stairs can take one stair or two stairs at a time. [4 points]
- (b) In how many ways can this person climb a flight of eight stairs? [2 points]
- (a) Let a_n be the number of ways to climb n stairs. If the person first climbs one stair, then there are a_{n-1} ways to climb the rest of the stairs; if the person first climbs two stairs, then there are a_{n-2} ways to climb the rest of the stairs. Hence $a_n = a_{n-1} + a_{n-2}$.
- (b) $a_1 = 1$ and $a_2 = 2$, so using the recurrence relation, $a_3 = 3$, $a_4 = 5$, $a_5 = 8$, $a_6 = 13$, $a_7 = 21$, and $a_8 = 34$.